For FM $\frac{1}{2\pi}\frac{d}{dt}\theta(t) = f(t) = f_c + k_f m(t)$

5.2 FM and PM

Definition 5.15. ***Frequency modulation*** (***FM***):

$$x_{\text{FM}}(t) = A\cos\Bigg(\overbrace{2\pi f_c t + \phi + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau}^{\theta(t)}\Bigg). \tag{69}$$

Its instantaneous frequency is

$$f(t) = f_c + k_f m(t).$$

5.16. ***Phase modulation*** (***PM***): The phase-modulated signal is defined in Definition 5.3 to be

$$x_{\text{PM}}(t) = A\cos(\overbrace{2\pi f_c t + \phi + k_p m(t)}^{\theta(t)})$$

Its instantaneous frequency is

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t) = f_c + \frac{k_p}{2\pi}\frac{d}{dt}m(t) \tag{70}$$

Therefore, the instantaneous frequency of the PM signal varies in proportion with the slope of $m(t)$.

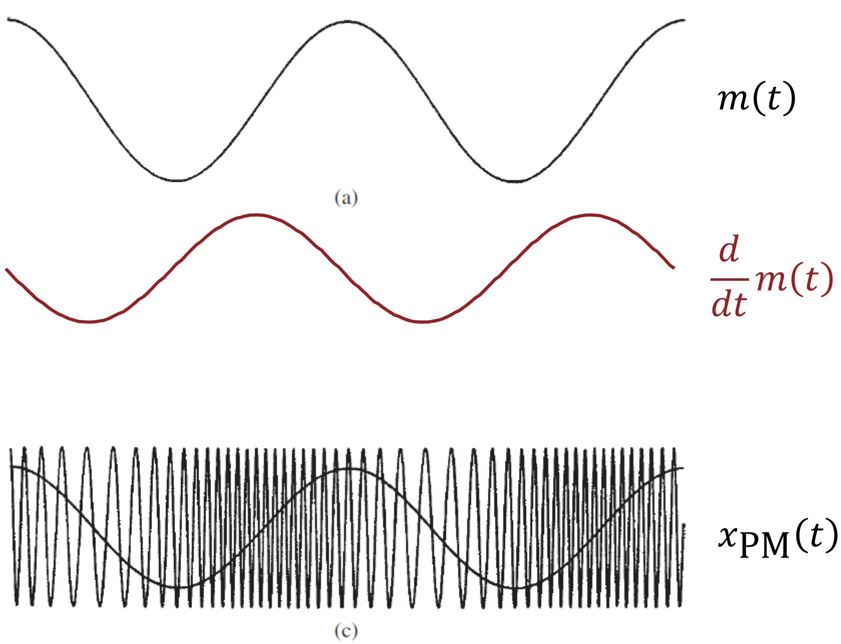

Figure 32: A revisit of the PM signal in Figure 29.

Analog modulation

(continuous wave)

Carrier : $A \cos(2\pi f_c t + \phi)$

AM : $A \rightarrow A + m(t)$

FM : $f(t) = f_c + k_f m(t)$

PM : $\phi \rightarrow \phi + k_p m(t)$

- continuous and differentiable
  $f(t) = f_c + \frac{k_p}{2\pi} \dot{m}(t)$
- discontinuous, have jump(s)
  the sudden change in the phase is proportional to the jump in $m(t)$.

In particular, the instantaneous frequency of the PM signal is maximum when the slope of $m(t)$ is maximum and minimum when the slope of $m(t)$ is minimum.

Example 5.17. Sketch FM and PM waves for the modulating signal $m(t)$ shown in Figure 33a.

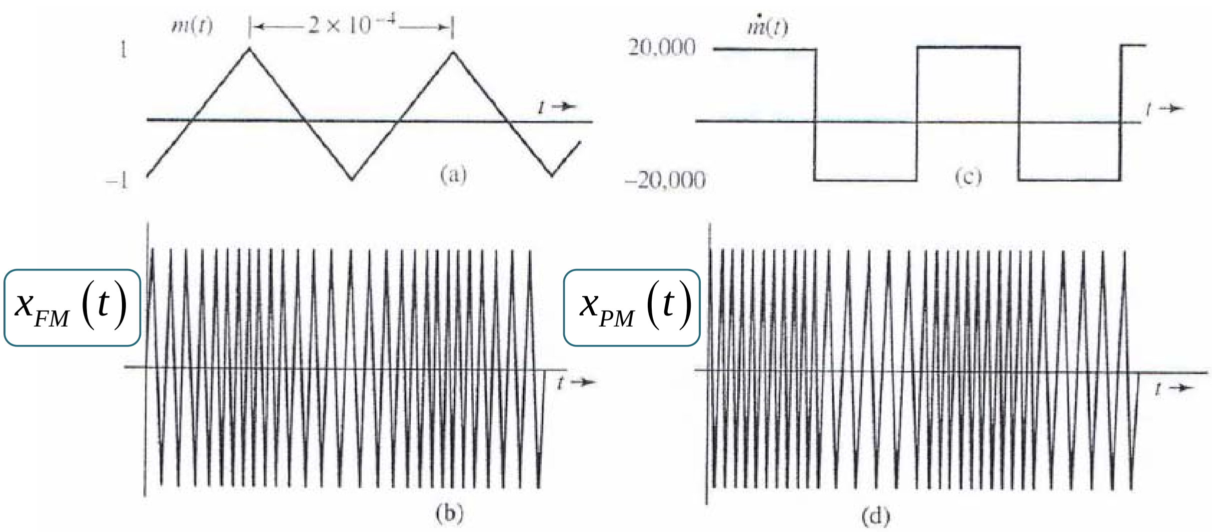

Figure 33: FM and PM waveforms generated from the same message.

Example 5.18. Sketch FM and PM waves for the modulating signal $m(t)$ shown in Figure 34a.

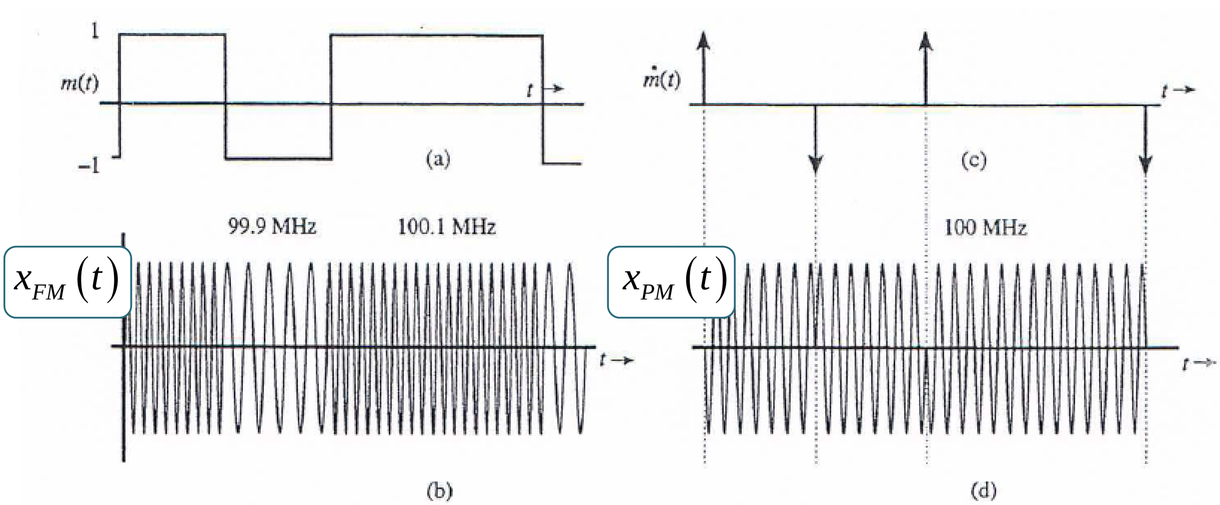

Figure 34: FM and PM waveforms generated from the same message.

The "indirect" method of sketching $x_{PM}(t)$ (using $\dot{m}(t)$ to frequency-modulate a carrier) works as long as $m(t)$ is a continuous signal. If $m(t)$ is discontinuous, this indirect method fails at points of discontinuities. In such a case, a direct approach should be used to specify the sudden phase changes.

5.19. Relationship between FM and PM:

- Equation (69) implies that one can produce frequency-modulated signal from a phase modulator.
- Equation (70) implies that one can produce phase-modulated signal from a frequency modulator.
- The two observations above are summarized in Figure 35.

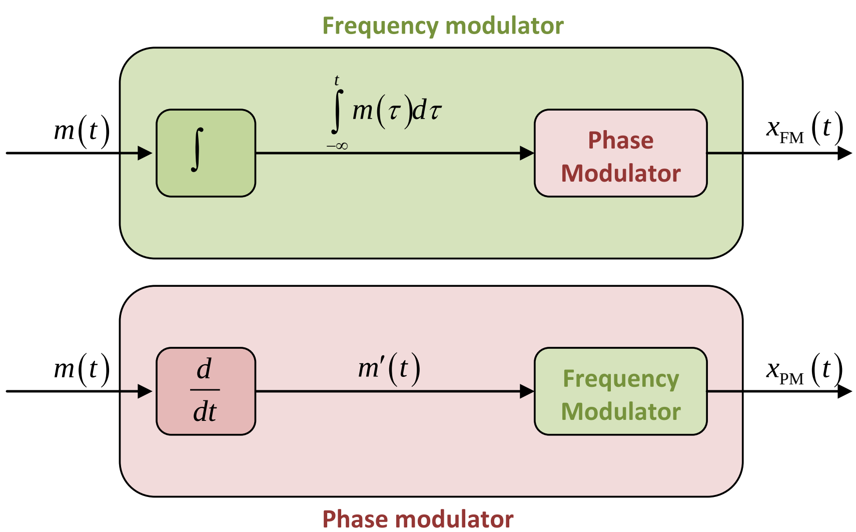

Figure 35: With the help of integrating and differentiating networks, a phase modulator can produce frequency modulation and vice versa [4, Fig 5.2 p 255].

- By looking at an angle-modulated signal $x(t)$, there is no way of telling whether it is FM or PM.
  - Compare Figure 29c and 29d in Example 5.6.
  - In fact, it is meaning less to ask an angle-modulated wave whether it is FM or PM. It is analogous to asking a married man with children whether he is a father or a son. [5, p 255]

5.20. ***Generalized angle modulation*** (or ***exponential modulation***):

$$x(t) = A\cos\left(2\pi f_c t + \theta_0 + (m * h)(t)\right)$$

where h is causal.

(a) ***Frequency modulation*** (***FM***): $h(t) = 2\pi k_f 1[t \geq 0]$

(b) ***Phase modulation*** (***PM***): $h(t) = k_p \delta(t)$.

5.21. So far, we have spoken rather loosely of amplitude and phase modulation. If we modulate two real signals $a(t)$ and $\phi(t)$ onto a cosine to produce the real signal $x(t) = a(t)\cos(\omega_c t + \phi(t))$, then this language seems unambiguous: we would say the respective signals amplitude- and phase-modulate the cosine. But is it really unambiguous?

The following example suggests that the question deserves thought.

Example 5.22. [8, p 15] Let's look at a "purely amplitude-modulated" signal

$$x_1(t) = a(t)\cos(\omega_c t).$$

Assuming that $a(t)$ is bounded such that $0 \leq a(t) \leq A$, there is a well-defined function

$$\theta(t) = \cos^{-1}\left(\frac{1}{A}x_1(t)\right) - \omega_c t.$$

Observe that the signal

$$x_2(t) = A\cos\left(\omega_c t + \theta(t)\right)$$

is exactly the same as $x_1(t)$ but $x_2(t)$ looks like a "purely phase-modulated" signal.

5.23. Example 5.22 shows that, for a given real signal $x(t)$, the factorization $x(t) = a(t)\cos(\omega_c t + \phi(t))$ is not unique. In fact, there is an infinite number of ways for $x(t)$ to be factored into "amplitude" and "phase".